CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY,
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MHD stagnation point flow of an upper-convected Maxwell fluid through porous media using Cattaneo-Christov heat flux model

by

Zaib-Un-Nisa

A thesis submitted in partial fulfillment for the
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CERTIFICATE OF APPROVAL

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Zaib-Un-Nisa

MMT151009

THESIS EXAMINING COMMITTEE

S No	Examiner	Name	Organization
(a)	External Examiner	Dr. Muhammad Sabeel Khan	IST, Islamabad
(b)	Internal Examiner	Dr. Rashid Ali	CUST, Islamabad
(c)	Supervisor	Dr. Muhammad Sagheer	CUST, Islamabad

Dr. Muhammad Sagheer
Thesis Supervisor
July, 2017

Dr. Muhammad Sagheer
Head
Department of Mathematics
Dated : July, 2017

Dr. Muhammad Abdul Qadir
Dean
Faculty of Computing
Dated : July, 2017

Certificate

This is certify that **Zaib-Un-Nisa**, Reg. NO. MMT151009 has incorporated all sugges tion, clarification and explanation by the thesis supervisor Dr. Muhammad Sagheer as well as external evaluator and internal examiner at Capital university of science and technology Islamabad. The research work titled her Thesis is: **MHD stagnation point flow of an upper-convected Maxwell fluid through porous media using Cattaneo-Christov heat flux model**.

Forwarded for necessary action

____

Dr. Muhammad Sagheer

(Thesis supervisor)

Declaration of Authorship

I, Zaib-Un-Nisa, declare that this thesis titled, 'MHD stagnation point flow of an upper-convected Maxwell fluid through porous media using Cattaneo-Christov heat flux model' and the work presented in it is my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Zaib-Un-Nisa

MMT151009

Signed: ____________________

Date: ____________________

"Education is not the learning of facts, but the training of mind to think."

Albert Einstein

CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY, ISLAMABAD

Abstract

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In this thesis, we study the magnetohydrodynamics stagnation point flow for the upper-convected Maxwell fluid with the thermal radiation and Joule heating effects using the Cattaneo-Christov heat flux model. The governing equations for velocity, temperature and concentration are in the form of nonlinear partial differential equations. Similarity transformations are used to convert the fundamental partial differential equations into a system of nonlinear ordinary differential equations. The resulting nonlinear ordinary differential equations are then solved by using the shooting method and the obtained results are compared with those obtained by the MATLAB built-in routine bvp4c. The effects of different parameters such as magnetic parameter, Prandtl number, Eckert number, radiational parameter, Deborah number, non dimensional thermal relaxation time parameter, Schmidt number on velocity, temperature and concentration profiles are illustrated by graphs and tables.

Acknowledgements

In the name of Allah, the Most Gracious and the Most Merciful Alhamdulillah, all praise is due to Allah; we praise Him, seek His help, and ask for His forgiveness. I am thankful to Allah, who supplied me with the courage, the guidance, and the love to complete this research. Also, I cannot forget the ideal man of the world and most respectable personality for whom Allah created the whole universe, Prophet Mohammed (Peace Be Upon Him).

Foremost, I would like to express my sincere gratitude to my supervisor **Dr. Muhammad Sagheer** for his passionate for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I would like to acknowledge CUST to providing me such a favourable environment to conduct this research.

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My heartiest and sincere salutations to my **Parents and Friends** who put their greater efforts in making me a good human being. I also feel grateful to **Saba Khan, Sana Khan, Hina Khan, M. Sohrab Khan, M. Afaq Khan, M. Ikram Khan**, who never let me down and always fortified me throughout the hard period of my research work.

May Almighty Allah shower His choicest blessings and prosperity on all those who assisted me in any way during completion of my thesis.

Zaib-Un-Nisa

Contents

List of Figures

List of Tables

Nomenclature

t	**time**
p	**pressure**
V	**velocity**
F	**force**
B_0	**magnetic induction**
k	**thermal conductivity**
k^*	**permeability**
K	**porousity parameter**
Ec	**Eckert number**
f	**dimensionless velocity**
M	**magnetic parameter**
A	**stretching ratio parameter**
C_p	**specific heat**
Pr	**Prandtl number**
Nu	**Nusselt number**
q_r	**radiative heat flux**
q_w	**wall heat flux**
Rd	**radiational parameter**
C_f	**skin friction coefficient**
T	**temperature**
T_∞	**free stream temperature**
T_w	**wall temperature**
U_∞	**free stream velocity**
(u, v)	**velocity components**
(x, y)	**cartesian coordinates**

Sc	**Schmidt number**
ρ	**fluid density**
μ	**dynamic viscosity**
ν	**kinematic viscosity**
τ	**stress tensor**
κ	**thermal conductivity**
α	**thermal diffusivity**
C_w	**constant concentration on wall**
C_∞	**ambient concentration**
η	**dimensionless similarity variable**
ψ	**stream function**
θ	**dimensionless temperature**
δ	**velocity slip parameter**
β	**thermal slip parameter**
σ	**electric conductivity parameter**
σ^*	**Stefan-Boltzmann constant**
γ	**Deborah number**
ϕ	**dimensionless concentration**
λ_1	**relaxation time of fluid**
λ_2	**relaxation time of heat flux**

DEDICATION

I dedicate this sincere effort to my dear ***Parents, Husband*** *and my elegant* ***Teachers*** *who are always source of Inspiration for me and their contributions are uncounted. I also dedicate this work to my* ***Father-in-law*** *who has been a constant source of support and encouragement during my research work.*

Chapter 1

Introduction

"The point in the flow field where the fluid's velocity is zero is called stagnation point". The study of viscous, incompressible, fluid past a permeable plate or sheet has great importance in the field of fluid dynamics. During the past few decades, the work on stagnation point flow of an incompressible fluid past a permeable sheet has got significant importance because of its large number of applications in manufacturing industries. Some of the main applications are refrigeration of electrical gadgets by fan, atomic receptacles cooling for the duration of emergency power cut, solar receiver, etc. The study of two dimensional (2D) stagnation point flow was first investigated by Hiemenz [1], whereas for getting the accurate solution, Eckert [2] extended this problem by adding the energy equation. In view of that Mahapatra and Gupta [3], Ishak et al. [4], and Hayat et al. [5] have studied the effects of heat transfer in stagnation point over a permeable plate.

"The study of magnetic properties of electrically conducting fluids is known as Magnetohydrodynamics (MHD). The study of MHD fluid flow was first introduced by Swedish Physicist, Alfven [6]". The investigation of MHD flow past a heated surface has received considerable attention because of its great applications in engineering problems like petroleum industries, MHD power generators, crystal growth etc. In recent years, mass and heat transfer on time dependent MHD natural convection flow of rotating fluid past a permeable sheet with effects of heat transfer and radiation was examined by Mbeledogu and Ogulu [7]. Kesvaiah et al. [8] investigated the time dependent MHD convective flow past a semi-infinite vertical permeable plate. While the analytical study of MHD heat and mass transfer oscilatory flow of micropolar fluid past a porous plate or sheet investigated by Modather et al. [9]. "Mabood et al. [10] investigated the MHD flow of viscous fluid in the existence of transpiration and furthermore detailed the interaction of chemical reaction". The impact of convective heat transfer in MHD flow of

Jeffrey fluid model over a permeable plate is reported by Hayat et al. [11]. Mustafa et al. [12] inspected the MHD flow of Maxwell fluid with convective heat transfer.

The study of flow and heat transfer generated by means of stretching medium has plenty of significance in numerous industrialized developments, (e.g, in the process of rubber and plastic sheets manufacturing, upgrading the solid materials like crystal, turning fibers etc). The most widely used coolant liquid among them is water. In above cases, flow and heat transfer investigation is of major importance because final product quality be determined to bulk level on the basis of coefficient of skin friction and heat transfer surface rate. Numerous investigators talked over different traits of stretching flow problem. Some of them are Crane [13], Chaim [14], Liao and Pop [15], Khan and Sanjayanand [16], and Fang et al. [17].

It is a well known fact that the phenomenon of heat transfer occurs between two bodies (or within the same body) due to the difference of temperature. In various industrial and engineering processes, the characteristics of heat transfer have huge demands in microelectronics, transportation and fuel cells etc. For the prediction of heat transfer analysis in various practical conditions, heat conduction law was suggested by Fourier [18], but it has a limitation that for the temperature field it generates a parabolic energy equation. To resolve this issue in classical Fourier law of heat conduction Cattaneo [19] added the thermal relaxation time. After that, "Christov [20] changed the Cattaneo law by time derivative in Maxwell-Cattaneo's model with Oldroyd's upper-convected derivative to conserve material-invariant formulation. Straughan [21] used Cattaneo-Christov model just to investigate thermal convection in an incompressible flow". Tibulle and Zampali [22] examined the uniqueness of Cattaneo-Christov heat flux model for flow of an incompressible fluid. Khan et al. [23] numerically investigated the Cattaneo-Christov heat flux model in viscoelastic flow due to exponentially stretching sheet.

The key feature for which the industrial product's worth/quality can be attained and the conserving ratio is made controllable is the magnetic field. For the purpose of observing the flow past a porous medium/sheet under different circumstances of MHD, several inquiries were made. Regarding this, Hayat et al. [24] and Pavlov [25] examined the impacts of magnetic field.

Currently, in the fields of engineering and fluid science, heat transfer and boundary layer flow of nano-fluid are the thrust areas of research. Many researchers examined the convective boundary layer flow of nano-fluid past a stretched sheet e.g, [26–30].

In future, advancement in nano-technology is expected for making unbelievable changes in our lives. A very big number of researchers are working in this area due to its great use in the engineering and its linked areas. In the process of air cleaning, development of microelectronics, safety of nuclear reactors etc, heat and mass transfer of thermophoretic magnetohydrodynamic flow consumes prospective uses. Choi [31] was the first who introduced the idea of "nanofluids" and presented the report on the heat transfer properties of nano-fluids. The thorough explosure on themophoretic flow was examined by Derjaguin and Yalamov [32]. Heat and mass transfer of MHD thermophoretic stream above plane surface was also studied by Issac and Chamka [33]. Thermophoresis effect on aerosol particles was investigated by Tsai [34].

In fluid temperature, no doubt, viscous dissipation produces a considerable ascend. This would happen because of change in kinetic motion of fluid into thermal energy. Viscous dissipation is unavoidable in case of flow field in high gravitational field. Viscous flow past a nonlinearly stretching sheet was deliberated by Vajravelu [35]. For external natural convention flow over a stretching medium, the effect of viscous dissipation was also studied by Mollendroff and Gebhart [36], whereas the impact of Joule heating and viscous dissipation on the forced convection flow with thermal radiation was presented by Duwairi [37].

Thesis contribution:

In this thesis we provide a review study of Shah et al. [38] and extend the flow analysis by considering the additional effects of stagnation point and concentration equation. In this work, the governing set of partial differential equations is converted into an arrangement of nonlinear coupled ordinary differential equations by utilizing appropriate similarity variables. The numerical solution has been found by using the numerical technique namely shooting method and then the numerical calculations are compared with those computed through the MATLAB builtin function bvp4c Elnashaie and Uhlig [39]. The numerical solutions are also discussed for different physical parameters graphically and tabularly.

Thesis outline:

The thesis is organised as follow:

In **Chapter 2**, some basic definitions and the relevant material is presented.

Chapter 3 contains a comprehensive review of Shah et al. [38]. "We consider the steady, laminar, incompressible, two-dimensional MHD flow of UCM fluid past a semi-infinite permeable sheet".

In **Chapter 4**, The work of Shah et al. [38] is extended by considering the additional effects of stagnation point with concentration equation. The reduced system of ordinary differential equations after applying a proper similarity transformation are solved numerically. Graphs and tables describe the behavior of physical parameters.

Chapter 5 summarizes the research work and gives the main conclusion occurring from the whole research and recommendations for the future work.

All the references used in this thesis are listed in **Bibliography**.

Chapter 2

Basic definitions and governing equations

This chapter contains explanation of basic definitions, concepts, governing laws, methods, terminologies which are helpful in the next chapters Genick BarMeir [40].

2.1 Basic Definitions

Definition 2.1.1. (Fluid) "A fluid is a material which has the ability to flow". Further, fluids are categorised into liquids and gases. Liquids take the shape of the container while gases do not.

Definition 2.1.2. (Fluid mechanics) "The old branch of physics that deals with the study of fluids in motion or in the state of rest". It is related with different fields such as biomedicine, physical chemistry, geophysics and also some branches of engineering are linked with it. It encompasses with fluid statics and fluid dynamics.

Definition 2.1.3. (Fluid statics) "In fluid statics, we study the behaviour of fluids at the state of rest". It is also referred to as hydrostatics.

Definition 2.1.4. (Fluid dynamics) "The branch of fluid mechanics that is concerned with motion of fluids from one place to another".

Definition 2.1.5. (Steady and unsteady flows) "Fluid flows can be classified as steady or unsteady on the basis of the fluid properties. The flow is said to be **steady**, if the fluid properties like velocity, density, etc. do not vary with time. Water flow with consistent release through a pipeline is an example of steady flow". Mathematically,

$$\frac{\partial B}{\partial t} = 0,$$

where B denotes any fluid property.
On the other hand "flow in which fluid properties change with time is known as **unsteady** flow". Water flow with varying release through a pipe is an example of unsteady flow. Mathematically, In this case,

$$\frac{\partial B}{\partial t} \neq 0.$$

Definition 2.1.6. (Laminar and turbulent flows) "A flow is known as **laminar** if the fluid flows in a regular manner, whereas the flow in which the fluid flows randomly is said to be **turbulent** flow. The most common example of both flows can be observed in the cigarette smoke".

Definition 2.1.7. (Compressible and incompressible flows) "The fluid flow type in which the density of the fluid is changed due to change in pressure is known as **compressible** flow". Generally, all gases are treated as compressible.

$$\frac{D\rho}{Dt} = 0,$$

where, ρ be the fluid's density and $\frac{D}{Dt}$ is the material derivative. Mathematically, material derivative is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla. \tag{2.1}$$

In above Eq.$\mathbf{V}$ indicates the fluid's velocity and ∇ is the differential operator. In Cartesian coordinate system ∇ can be written as

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}.$$

"The flow is known to be **incompressible** if the fluid's density remains constant". Liquids are treated as incompressible.

Definition 2.1.8. (Uniform and non-uniform flows) "The flow type in which, at all sections of the channel, fluid particles have equal velocity is known as **uniform flow**. For instance, flow through a long straight pipe of uniform diameter is uniform flow. Whereas if the fluid velocity is different at different points, then the flow is known as **non-uniform flow**. Flow through a long pipe with varying cross section is consider as non-uniform flow."

Definition 2.1.9. (Viscosity) "It is the resistance of the substance to flow. It depends upon the size and shape of molecules. It is related with the concept of shear force". Mathematically, it is denoted by μ. Some examples are honey, oil, sulfuric acid etc.

Definition 2.1.10. (Kinematic viscosity) "It is defined as the ratio of the dynamic viscosity μ to the density ρ of the fluid. It is also referred to as momentum diffusivity. It is denoted by Greek letter ν". Mathematically,

$$\nu = \frac{\mu}{\rho}.$$

In SI system of units the unit of kinematic viscosity is m^2/s and dimension is $[L^2T^{-1}]$.

Definition 2.1.11. (Dynamic viscosity) "It is defined as the tangential force per unit area necessary for exchanging one horizontal plane with respect to the other. Mathematically, it can be expressed as the ratio of shear stress to the rate of shear strain and is denoted by μ.

$$\text{Viscosity}(\mu) = \frac{\text{Shear stress}}{\text{Rate of shear strain}}.$$

In the above expression μ is called the coefficient of viscosity. It is also referred to as absolute viscosity or dynamic viscosity or simply viscosity having dimension $[ML^{-1}T^{-1}]$. Unit of viscosity in SI system is kg/ms or Pascal-second [Pa.s]".

Definition 2.1.12. (Newtonian and non-Newtonian fluid) "Those fluids which obey the Newton's law of viscosity are called **Newtonian fluids**. Water, air, oil are few examples of Newtonian fluids".

Mathematically, Newtonian's fluid behaviour is expressed by the following equation

$$\tau = \mu\frac{du}{dy}.$$

Where, in this equation μ is the viscosity, $\frac{du}{dy}$ is the deformation rate and τ is the stress tensor.
On the other hand, "fluids which do not obey Newton's law of viscosity are referred to as **non-Newtonian fluids.** Shampoo, toothpaste, blood and ketchup are the main examples of non-Newtonian fluids".

Definition 2.1.13. (Porosity) "It is defined as the proportion of pores volume (vacant space) to the mass volume of a permeable media". A permeable media generally recognized by its permeability. An example of porosity is the quality of a sponge. The momentum equation with MHD and porosity is defined as

$$\rho\frac{D\mathbf{V}}{Dt} = \nabla.\tau - \rho\sigma\mathbf{B^2}\mathbf{V} - \rho k\mathbf{V}. \tag{2.2}$$

Here **B** and k and are the magnetic field and porosity of the medium respectively.

Definition 2.1.14. (Thermal conductivity) "Thermal conductivity (κ) is the property of a material related to its capability to transmit heat". Mathematically,

$$\kappa = \frac{q\nabla l}{S\nabla T},$$

where q is the heat passing through a surface area S and causing a temperature difference ∇T over a distance of ∇l. Here l, S and ∇T all are assumed to be of unit measurement. In SI system of units the unit of thermal conductivity is $\frac{W}{m.\kappa}$ and its dimension is $[MLT^{-3}\theta^{-1}]$.

Definition 2.1.15. (Thermal diffusivity) "Thermal diffusivity is material property for characterizing unsteady heat conduction". Mathematically, it can be expressed as,

$$\alpha = \frac{\kappa}{\rho C_p},$$

where κ is the thermal conductivity of material, ρ represent the density and C_p be the specific heat capacity respectively. The unit and dimension of thermal diffusivity in SI system are m^2s^{-1} and $[LT^{-1}]$ respectively.

Definition 2.1.16. (Boundary layer flow) "The layer of fluid adjacent to the solid surface past which the fluid flows". In fluid mechanics, boundary layer flows play a significant role. The basic idea of boundary layer was first introduced by L. Prandtl. There are two types of boundary layer flow.

- Hydrodynamic (velocity) boundary layer
- Thermal boundary layer

Definition 2.1.17. (Hydrodynamic boundary layer) "It is an area of the liquid nearest to the solid surface, in which the flow pattern is specifically influenced by the viscous drag from the surface".

Definition 2.1.18. (Thermal boundary layer) "It is an area of the liquid nearest to the solid surface, where fluid temperature is directly influenced by the heating or cooling from the surface".

2.2 Basic equations

Definition 2.2.1. (Generalized continuity equation) We know that, mass conservation law states that "Mass of fluid can neither be created nor be destroyed". Continuity equation is the mathematical expression which expresses the mass conservation law. For compressible fluids, continuity Eq. can be written mathematically as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0. \tag{2.3}$$

where t is time, ρ is the fluid density and V is the velocity of the fluid. "If the fluid density is constant in this case, above Eq. is referred to as Eq. of continuity for incompressible fluids".

Mathematically,

$$\nabla \cdot \mathbf{V} = 0.$$

Definition 2.2.2. (Generalized momentum equation) "Every fluid particle obeys Newton's second law of motion which is at rest or in steady state or accelerated motion. This law states as the rate of change of momentum is equivalent to applied force". The mass of the framework is consistent, in this manner "Newton's second law can be composed as

$$m\frac{D\mathbf{V}}{Dt} = \mathbf{F}.$$

The flow of the fluid is represented by the differential equation as

$$\rho\frac{D\mathbf{V}}{Dt} = \nabla.\tau + \rho\mathbf{b},$$

where $\rho\mathbf{b}$ is the net body force, τ is the Cauchy stress tensor and $\nabla.\tau$ are the surface forces".

2.3 Heat transfer

It is basically, "the change in thermal energy of one medium, or media to another medium due to temperature difference". Medium may be two solids, a solid and a gas or liquid.

2.3.1 (Modes of heat transfer) The elementary modes of heat transfer are "conduction, convection and radiation".

Definition 2.3.2. (Conduction) "It is the transfer of energy such as an electric current or heat, through a substance. Heat conduction process depends on physical properties of the material, temperature gradient, length of the path and the cross-section of the material. In heat conduction process, heat energy is transferred and distributed from atom to atom and molecule to molecule within the substance by direct method".

Definition 2.3.3. (Convection) "When a heated fluid like gas or liquid, is forced to move far from the source, it takes the thermal energy with it. This kind of heat transfer is known as convection".

Definition 2.3.4. (Radiation) "It is the transmission of energy in the form of waves, that generates from source and travels through space". Radiation occurs through a vaccum or transparent medium. For example, "sunlight is the form of radiation that travels through space to our planet i.e., earth". As there is no fluid in space, convection heat transfer is not responsible for transfer of heat. Thus in this case, radiation brings heat to earth.

2.4 Dimensionless numbers

(i) Prandtl number (Pr)

"Prandtl number is a dimensionless number. It is expressed as the ratio of momentum diffusivity ν to thermal diffusivity α". Numerically, we can formulate it as:

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k},$$

where C_p denotes the specific heat, μ is the dynamic viscosity and κ stands for thermal conductivity.

(ii) Deborah number (γ)

"It is defined as the ratio of the relaxation time to deformation time", i.e.,

$$\gamma = \frac{\lambda V}{2x}.$$

(iii) Skin friction coefficient (C_f)

"Skin friction coefficient is the ratio between the fluid and the solid surface which measures the retardation of the fluid due to friction". Mathematically,

$$C_f = \frac{2\tau_w}{\rho V^2},$$

where τ_w denotes the wall shear stress, ρ denotes the density and V is the fluid velocity.

(iv) Eckert number (Ec)

A number with no dimensions used in continuum mechanics. It defines the "relationship between flow's kinetic energy and boundary layer difference". Mathematically,

$$Ec = \frac{V^2}{C_p \Delta T},$$

where V is the fluid velocity, C_p is the specific heat and ΔT is the difference of initial and final temperature.

(v) Schmidt number (Sc)

It is the "ratio between viscosity ν and molecular diffusion D". It is denoted by Sc and mathematically we can write it as,

$$Sc = \frac{\nu}{D}.$$

Definition 2.4.1. (Magnetohydrodynamics) "It also referred to as Magneto fluid dynamics or hydromagnetics. It deals with the study of magnetic properties of electrically conducting fluids. It is denoted by MHD".

Definition 2.4.2. (Stagnation point) "Stagnation point of the flow field is a point where the local velocity of the fluid is zero".

Definition 2.4.3. (Joule heating) "It is the process in which heat is generated by passing an electric current through a metal. Joule heating also referred to as resistive heating and ohmic heating".

Chapter 3

MHD effects and heat transfer for the UCM fluid along with Joule heating and thermal radiation using Cattaneo-Christov heat flux model

3.1 Introduction

In this chapter, we review a recently published article of Shah et al. [38]. In this article, time independent, incompressible, two-dimensional laminar and magnetohydrodynamics flow of an "upper-convected Maxwell fluid" past a semi-infinite porous plate is considered. The stretching sheet is assumed to have ambient temperature is T_∞ and constant temperature T_w. The heat flux model is introduced by Christov [41] has been considered. We reconstruct the flow equations and the obtained set of partial differential equations (PDEs) is then converted into an arrangement of nonlinear, coupled ordinary differential equations (ODEs) by utilising some reasonable similarity transformations. After this, the set of ordinary differential equations (ODEs) is solved by applying shooting method. Finally, the numerical results are discussed for different parameters and also a comparison of these numerical results with those computed by the MATLAB built-in routine bvp4c is presented. "MATLAB bvp4c is a finite difference code that uses a collocation method. Boundary value problems (BVP) for ordinary differential equations (ODEs) can be solved using MATLAB bvp4c solver. It starts solution with an initial guess supplied at an initial mesh points and changes step-size to get the specified accuracy Elnashaie and Uhlig [39]".

3.2 Mathematical modeling

Consider the two-dimensional MHD, laminar, incompressible and steady state flow of a fluid past a semi-infinite stretching sheet. The geometry of the flow model is given in Figure 3.1.

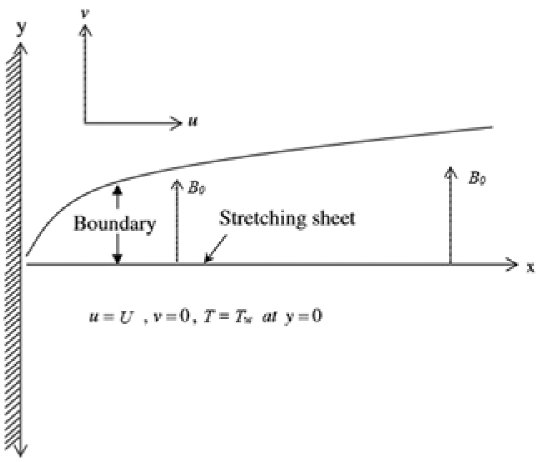

FIGURE 3.1: Geometry for the flow under consideration.

Here Cattaneo-Christov heat flux model is under consideration. Along y-axis, a constant magnetic field of strength B_0 is applied perpendicular to x-axis. Further its is supposed that the induced magnetic field is negligible. It is supposed that boundary layer approximations are appropriate to the governing equations considered by Renardy [42] for "Maxwell fluid models". By making use of boundary layer approximations, the arrangement of representing PDEs like continuity, momentum and energy with impacts of Joule heating and thermal radiation for the MHD flow of UCM fluid by neglecting the viscous dissipation and pressure gradient, can be expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}) = \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0^2}{\rho}u, \tag{3.2}$$

$$\rho C_p(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = -\nabla \cdot q + \sigma B_0^2 u^2 - \frac{\partial q_r}{\partial y}. \tag{3.3}$$

In the above equations u and v are the components of velocity along the x and y directions respectively. Moreover, λ_1 denotes the relaxation time, ρ denotes the fluid's density, B_0 is constant magnetic filed, σ be the electric conductivity constant, kinematic viscosity is denoted by ν, C_p is the specific heat, fluid temperature is T, q_r is the radiative heat flux. According to Christov [41], we have the following relation

$$q + \lambda_2 \left(\frac{\partial q}{\partial t} + V.\nabla q + (\nabla.V)q \right) = -k\nabla T, \tag{3.4}$$

On eliminating q from Eqs. (3.3) and (3.4), we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \lambda_2 \left(\begin{array}{c} (u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y})\frac{\partial T}{\partial x} + (u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y})\frac{\partial T}{\partial y} + \\ u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 T}{\partial x \partial y} \end{array} \right) = \alpha\frac{\partial^2 T}{\partial y^2} + \sigma\frac{B_0^2}{\rho C_p}u^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}, \tag{3.5}$$

where V denotes the fluid velocity, λ_2 is the relaxation time and thermal diffusivity is denoted by α. Also, the radiative heat flux is given by

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}. \tag{3.6}$$

"Expansion of T^4 about T_∞ by making use of Taylor's series is":

$$\text{“}T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + ...\text{”},$$

Ignoring the terms with higher order in $(T - T_\infty)$ we get,

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}, \tag{3.7}$$

where σ^* is Stefan-Boltzman constant and k^* is the absorption coefficient. The boundary conditions for the above system of partial differential equations are,

$$u = U, \quad v = 0, \quad T = T_w, \qquad \text{at} \quad y = 0, \tag{3.8}$$

$$u \to 0, \quad T \to T_\infty, \qquad \text{as} \quad y \to \infty. \tag{3.9}$$

Now, we introduce similarity transformations or (dimensionless variables) Shah et al. [38] which are useful in transforming the PDEs Eqs. (3.1) - (3.3) into the ODEs along with the boundary conditions Eqs. (3.8) - (3.9).

$$\eta = \sqrt{\frac{U}{\nu x}}(y), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad u = Uf'(\eta), \quad v = \frac{-1}{2}\sqrt{\frac{U\nu}{x}}(f - \eta f'), \tag{3.10}$$

where the prime represents derivative w.r.t η, T_∞ and T_w are the ambient and constant fluid temperature at wall respectively and θ is the dimensionless temperature. The set of corresponding ODEs is:

$$f''' + \frac{1}{2}ff'' - \frac{\beta}{2}(\eta f'^2 f'' + 2ff'f'' + f^2 f''') - Mf' = 0, \tag{3.11}$$

$$\frac{1}{Pr}(1 + \frac{4}{3}Rd)\theta'' + \frac{1}{2}f\theta' - \frac{\gamma}{2}(3ff'\theta' + f^2\theta'') + MEcf'^2 = 0. \tag{3.12}$$

The boundary conditions for the governing ODEs are

$$f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1, \quad \text{at} \quad \eta = 0, \tag{3.13}$$

$$f'(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \text{at} \quad \eta \to \infty. \tag{3.14}$$

In Eqs. (3.11) - (3.12), β is the Deborah number, Pr is the Prandtl number, M is the magnetic parameter, radiational parameter is Rd, Ec is the Eckert number and γ is the non dimensional thermal relaxation time parameter. Some important dimensionless parameters are formulated as

$$\beta = \frac{\lambda_1 U}{2x}, \quad Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}, \quad M = \frac{\sigma {B_0}^2 x}{\rho U}, \quad Rd = \frac{4\sigma^* T_\infty^3}{kk^*}, \quad Ec = \frac{U^2}{c_p(T_w - T_\infty)} \quad \text{and} \quad \gamma = \frac{\lambda_2 U}{2x}.$$

3.3 Numerical solution

As system of Eqs. (3.11) - (3.14) with the associated boundary conditions is coupled and nonlinear, so approximate solution can not be found directly. For this we use the numerical technique i.e., the shooting method to find the approximate solution. By making use of this technique, we convert the system of higher order ODEs into the system of first order ODEs.

$$f''' = \frac{1}{2 - \beta f^2}(\eta\beta f'^2 f'' + 2\beta ff'f'' - ff'' + 2Mf'), \tag{3.15}$$

$$\theta'' = \frac{3Pr}{6 + 8Rd - 3Pr\gamma f^2}(3\gamma ff'\theta' - f\theta' - 2MEcf'^2). \tag{3.16}$$

subject to boundary conditions

$$f(\eta) = 0, \quad f'(\eta) = 1, \quad \text{at} \quad \eta = 0; \quad f'(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \tag{3.17}$$

$$\theta(\eta) = 1, \quad \text{at} \quad \eta = 0; \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \tag{3.18}$$

Let us denote

$$f = y_1, \quad f' = y_2, \quad f'' = y_3, \quad \theta = y_4, \quad \theta' = y_5. \tag{3.19}$$

The system of first Order ODEs along with the boundary conditions becomes

$$y_1' = y_2 \qquad y_1(0) = 0 \tag{3.20}$$

$$y_2' = y_3 \qquad y_2(0) = 1 \tag{3.21}$$

$$y_3' = \frac{1}{2 - \beta {y_1}^2}(\eta\beta {y_2}^2 y_3 + 2\beta y_1 y_2 y_3 - y_1 y_3 + 2M y_2) \qquad y_3(0) = s \tag{3.22}$$

$$y_4' = y_5 \qquad y_4(0) = 1 \tag{3.23}$$

$$y_5' = \frac{3Pr}{6 + 8Rd - 3Pr\gamma y_1^2}(3\gamma y_1 y_2 y_5 - y_1 y_5 - 2MEc y_2^2) \qquad y_5(0) = t. \tag{3.24}$$

For solving above system numerically, we replace the domain $[0, \infty)$ by the bounded domain $[0, \eta_\infty]$ where η_∞ is some suitable real number. In the above system of equations we have $y_3(\eta)$ and $y_5(\eta)$ at $\eta = 0$ i.e., s and t are missing conditions and are to be chosen such that

$$y_2(\eta_\infty, s, t) \approx 0 \qquad \text{and} \qquad y_4(\eta_\infty, s, t) \approx 0.$$

3.4 Results and discussion

This section aims to investigate the numerical impacts of different parameters such as Prandtl number Pr, non dimensional thermal relaxation time parameter γ, Deborah numbers β, Eckert number Ec, magnetic parameter M and radiational parameter Rd displayed graphically and tablularly. The computations are worked out for different values of the effects of magnetic parameter M, Eckert number Ec, Prandtl number Pr, Deborah number β and non dimensional thermal relaxation time parameter γ and also discussed the effect of the physical parameters on velocity and temperature profile.

The impact of different parameters like, magnetic parameter, radiational parameter, Eckert number, Prandtl number, radiational parameter is discussed graphically. In Table 3.1 and 3.2 numerical values for temperature gradient $-\theta'(0)$ and velocity $-f''(0)$ are calculated for different physical parameters.

						$-\theta(0)$	
Pr	γ	β	M	Ec	Rd	Shooting	bvp4c
0.72	0.5	0.5	0.1	0.1	0.23	0.3462925	0.3462925
0.3						0.26574360	0.265743
0.5						0.30250348	0.3025034
0.7						0.34220604	0.3422060
	0.2					0.3802616	0.3802614
	0.3					0.3689130	0.3689130
	0.4					0.3575874	0.3575875
		0.2				0.34510898	0.3451080
		0.5				0.34629247	0.3462925
		0.7				0.3469356	0.3469357
			0.3			0.3303649	0.3303691
			0.5			0.3175665	0.3175663
			0.7			0.3068422	0.3068427
				0.5		0.3266428	0.3266429
				0.9		0.3069934	0.3069933
				1.2		0.2922566	0.2922562
					0.3	0.3605506	0.3605506
					0.7	0.4437890	0.4437900
					1.8	0.6809324	0.6809437

TABLE 3.1: Numerical results of $-\theta'(0)$ for different values of Pr, γ, β, M, Ec and Rd.

						$-f''(0)$	
Pr	γ	β	M	Ec	Rd	Shooting	bvp4c
0.72	0.5	0.2	0.1	0.1	0.23	0.5169288	0.5169288
		0.5				0.4822495	0.4822495
		0.7				0.45824237	0.4582423
			0.1			0.4822495	0.4822495
			0.3			0.6450524	0.6450524
			0.5			0.7803249	0.7803250
			0.7			0.8972758	0.8972756

TABLE 3.2: Numerical results of $-f''(0)$ for $Pr = 0.72$, $\gamma = 0.5$, $Ec = 0.1$ and $Rd = 0.1$.

For visualising the effects of different parameters on velocity $f'(\eta)$ and temperature profile $\theta(\eta)$, graphs are plotted below. In every one of these estimations, we have considered $\gamma = 0.5$, $Pr = 0.72$, $M = 0.1$, β= 0.5, $Rd = 0.23$ and $Ec = 0.1$. Figure 3.2 determines the impact of magnetic parameter M on dimensionless velocity $f'(\eta)$. The graphical demonstration shows that for the increasing values of magnetic parameter M, there is decrease in the velocity profile. It happens for the reason that Lorentz force which decreases the horizontal flow risen by rising the magnetic parameter M. Figure 3.3 is the graphical representation which shows the temperature profile for the various values of magnetic parameter M. By this graph, it is observed that the effect of magnetic parameter M on velocity and temperature profile is opposite. From Figure 3.4, it can be seen that by increasing the value of Eckret number Ec, temperature profile also increases. The effect of radiational parameter Rd on dimensionless temperature θ is represented in Figure 3.5 . In this graph it is observed that on increasing the value of radiational parameter Rd, temperature profile θ also increases. So, rate of heat transfer decreases with increase in radiational parameter Rd, because of that temperature profile increases. In Figure 3.6, the influence of non dimensional thermal relaxation time parameter γ on temperature profile θ is shown. This graph represents that on increasing the non dimensional thermal relaxation time parameter γ, value of temperature profile θ decreases, because of this fact that when non dimensional thermal relaxation time parameter γ increases results decreases in time of deformation which causes the decrease in temperature of fluid. Figure 3.7 shows the influence of Doberah number β on velocity profile f'. For the increasing values of Deborah number β, velocity increases near the plate while in the rest portion of the boundary layer it diminishes for expanding β. From Figure 3.8, it can be seen that by the increase in Deborah number β, temperature profile θ is increased. Figure 3.9 illustrates the difference of temperature θ for different values of the Prandtl number Pr. It is perceived that the temperature decreases, for the increasing values of Prandtl number. Decrease in thermal boundary layer comes across when Pr is larger and decrease in the thermal diffusivity causes rise in the Prandtl number. In this way increment in Pr diminishes diffusivity and the variety in thermal characteristics increments.

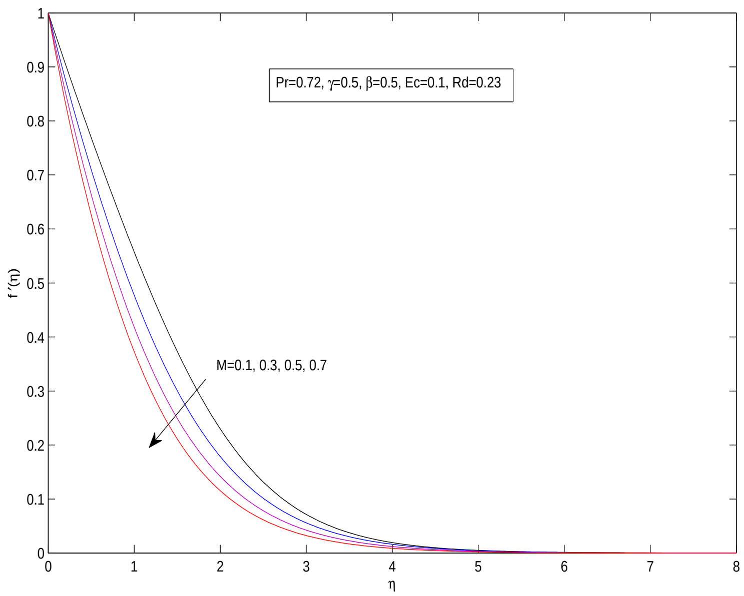

FIGURE 3.2: Influence of M on the dimensionless velocity f'

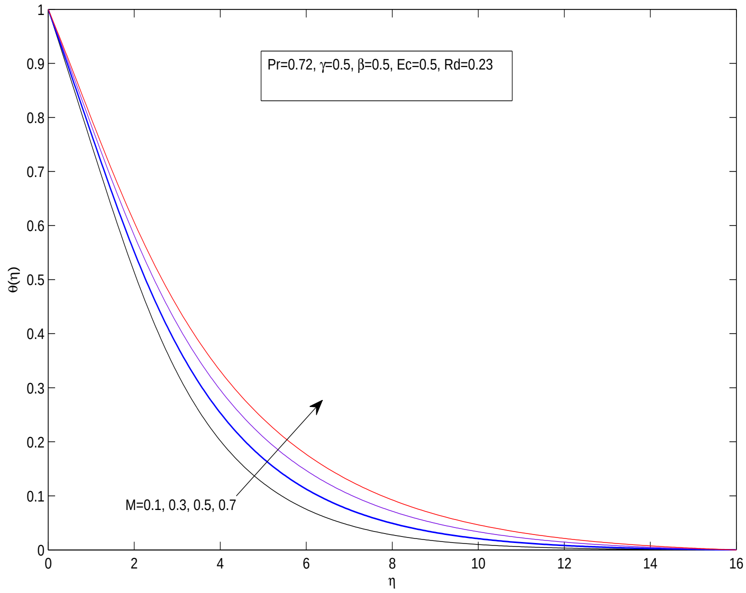

FIGURE 3.3: Influence of M on θ.

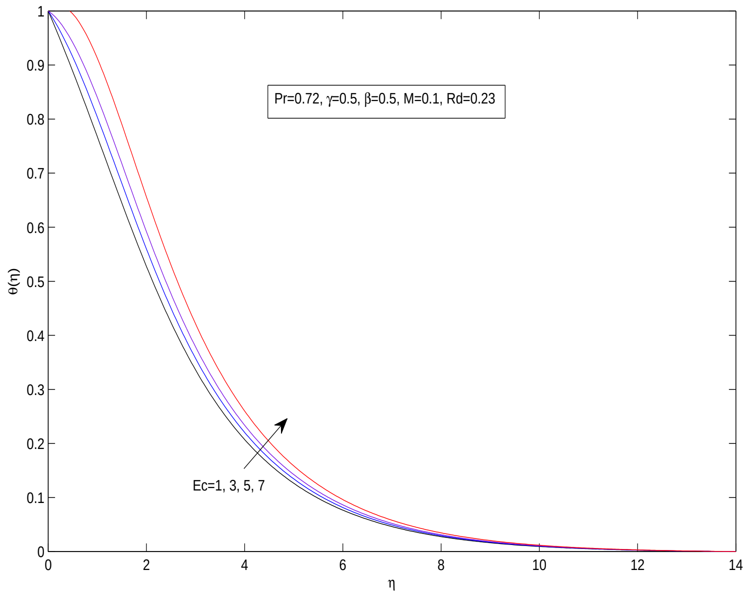

FIGURE 3.4: effect of Ec on the θ.

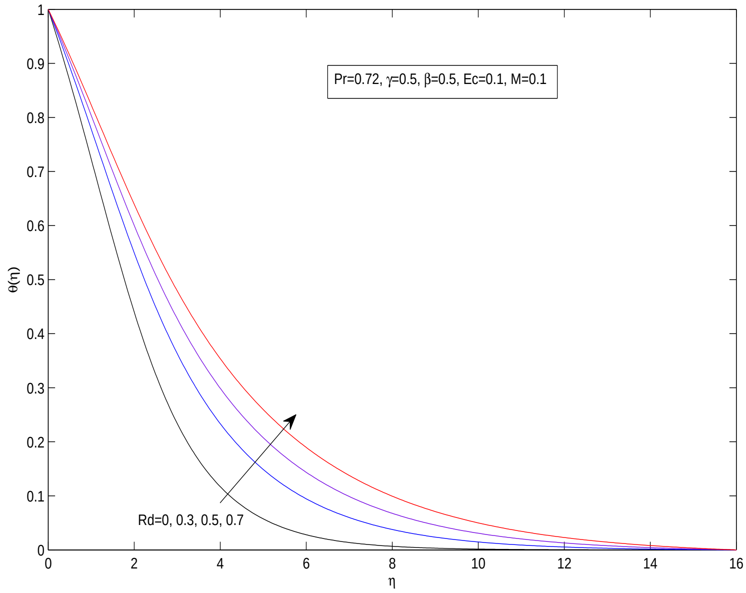

FIGURE 3.5: Effect of Rd on the dimensionless temperature θ.

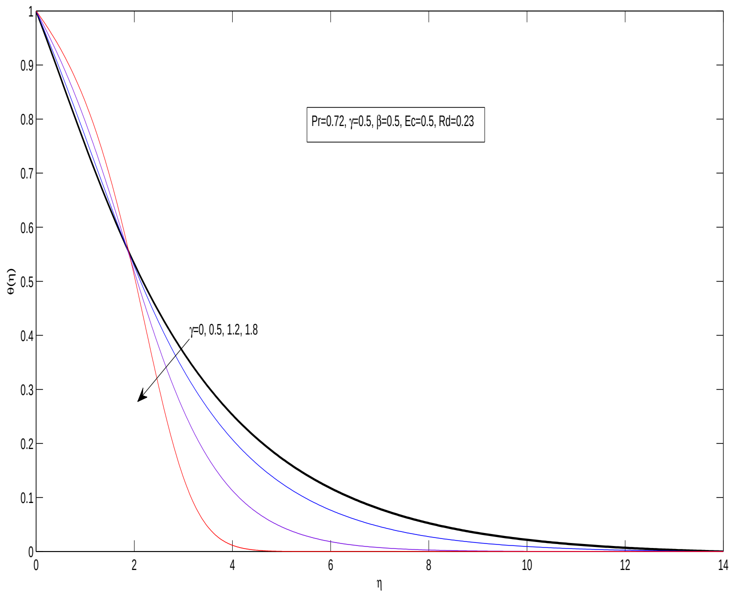

FIGURE 3.6: Impact of γ on the dimensionless temperature θ.

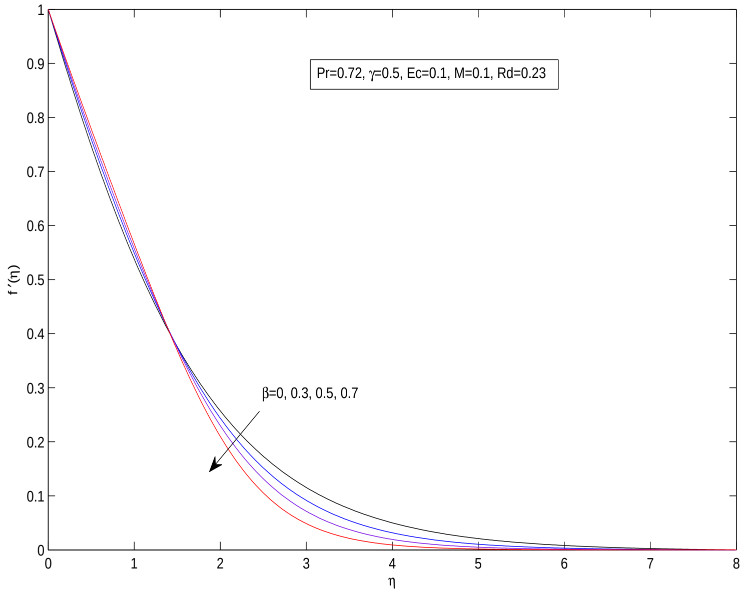

FIGURE 3.7: Impact of β on dimensionless Velocity $f\prime$.

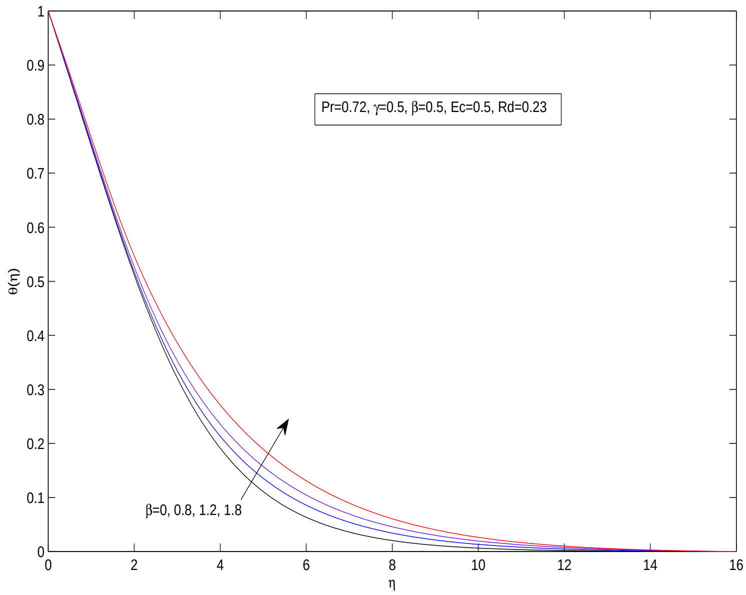

FIGURE 3.8: Effect of β on the dimensionless temperature θ.

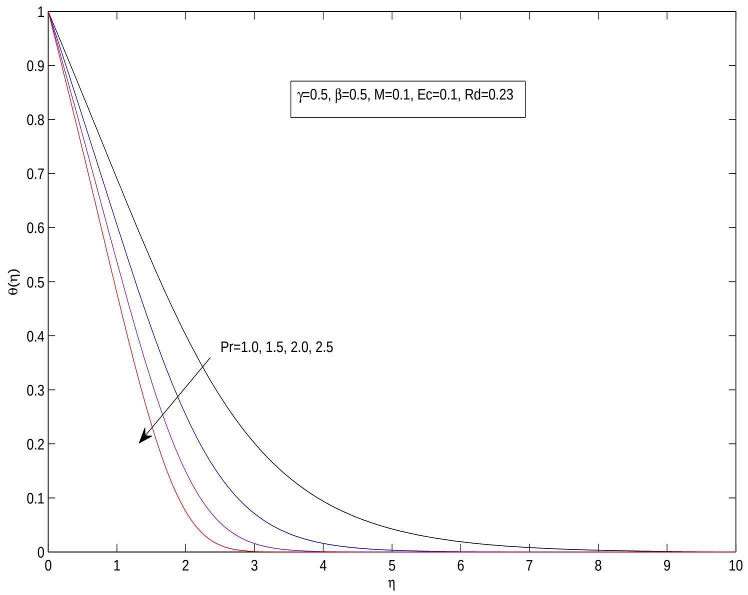

FIGURE 3.9: Impact of Pr on the dimensionless Temperature θ.

Chapter 4

MHD stagnation point flow with heat and mass transfer past a porous sheet

4.1 Introduction

In this chapter we extend the flow model of Shah et al. [38] presented in previous chapter. We will examine steady, incompressible, laminar, 2D magnetohydrodynamics (MHD) stagnation point flow with concentration past a permeable plate. The stretching plate has constant temperature T_w and ambient temperature is T_∞ also C_w is the constant concentration on wall and C_∞ be the ambient concentration. The nonlinear partial differential equations of velocity, temperature and concentration are converted into a system of ordinary differential equations (ODEs) by using helpful similarity transformations. Numerical solution of these governing ordinary differential equations (ODEs) is obtained by using shooting technique. Finally, the numerical results are discussed for different physical parameters affecting flow and heat transfer and found to be in excellent agreement with those computed by the MATLAB built-in routine bvp4c Elnashaie and Uhlig [39]. Effect of various physical parameters on dimensionless velocity, temperature and concentration are explained by graphs and tables.

4.2 Problem formulation

Considered the laminar, two-dimensional, steady and MHD stagnation point flow of a fluid with effects of heat transfer past a porous medium. It is assumed that the fluid is taken as viscous and incompressible. The related set of equations like continuity, momentum energy and concentration with the corresponding boundary conditions are given in Eqs. (4.1) - (4.6).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{4.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y})=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k_0}u \tag{4.2}$$

$$-\sigma\frac{B_0^2}{\rho}(u-u_\infty+\lambda_1 v\frac{\partial u}{\partial y})+u_\infty\frac{du_\infty}{dx},$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_2\left[\begin{array}{c}(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})\frac{\partial T}{\partial x}+(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})\frac{\partial T}{\partial y}+\\ u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 T}{\partial x}\partial y\end{array}\right]=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{4.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}. \tag{4.4}$$

From Eqs. (4.1) - (4.6), u and v are the components of velocity along x and y direction respectively.Furthermore, fluid's relaxation time is denoted by λ_1, whereas λ_2 is the relaxation time of the heat flux. T is the temperature and ρ is the density of fluid. The parameter k_0 is taken as constant and represents the permeability of permeable medium. Also, U_∞ is the velocity of the stagnation point flow. B_0 is the constant magnetic field, σ is the constant of electrical conductivity. α is the thermal conductivity constant and C is the concentration field. The associated boundary conditions for the above system of equations are,

$$u=ax,\quad v=0,\quad T=T_w,\quad C=C_w,\qquad \text{at}\quad y=0, \tag{4.5}$$

$$u\to U_\infty=cx,\quad T\to T_\infty,\quad C\to C_\infty,\qquad \text{as}\quad y\to\infty. \tag{4.6}$$

Now, we introduce similarity transformations or (dimensionless variables) Noor Fadiya Mohd Noor [43] which are useful in transforming the PDEs (4.1) - (4.4) into the ODEs along with the boundary conditions (4.5) and (4.6)

$$\eta=\sqrt{\frac{a}{\nu}}y,\quad u=axf'(\eta),\quad v=-\sqrt{a\nu}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ \ and\ \ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \tag{4.7}$$

where the prime represents derivative w.r.t η, T_∞ and T_w are the ambient and constant fluid temperature at wall respectively, θ is the dimensionless temperature and C_w is the constant concentration and C_∞ is the ambient concentration .

The differential Eqs. (4.1) - (4.4) with the associated boundary conditions (4.5) and (4.6) take the following form after applying the similarity transformation

$$f''' + (1 + M\beta)ff'' - f'^2 + \beta(2ff'f'' - f^2f''') - (M + K)f' + MA + A^2 = 0, \quad (4.8)$$

$$\theta'' + Prf\theta' - \gamma Pr(ff'\theta' + f^2\theta'') = 0, \quad (4.9)$$

$$\phi'' + Scf\phi' = 0. \quad (4.10)$$

The corresponding boundary conditions for above set of ODEs are:

$$f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1, \quad \text{at} \quad \eta = 0, \quad (4.11)$$

$$f'(\eta) \to A, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0, \quad \text{at} \quad \eta \to \infty. \quad (4.12)$$

Where in the above system of Eqs. the prime denotes the derivative w.r.t η. In Eqs.(4.8) - (4.12), M is the magnetic field parameter, β is the Deboreh number, the parameter K represents the permeability of the porous medium, Pr is Prandtl number. It depends on the viscosity, thermal conductivity and specific heat of the fluid, A is the unsteady parameter and is known as stretching ratio parameter. Moreover, γ is the non dimensional thermal relaxation time parameter and Sc is the schmidt number.

$$M = \frac{\sigma B_0^2}{\rho a}, \quad P_r = \frac{\mu C_p}{k}, \quad K = \frac{\nu}{ak_0}, \quad \beta = \lambda_1 a, \quad \gamma = \lambda_2 a, \quad Sc = \frac{\nu}{D}.$$

4.3 Numerical solution

As system of Eqs. (4.8) - (4.10) with the associated boundary conditions Eqs. (4.11) and (4.12) is coupled and non-linear, so approximate solution can not be found directly. For this we use the numerical technique i.e., the shooting method to find the approximate solution. By making use of these technique, we convert system of higher order ODEs into set of first order ODEs.

$$f''' = \frac{1}{1 - \beta f^2}(-MA - A^2 + (M + K)f' + f'^2 - 2\beta ff'f'' - (1 + M\beta)ff''), \quad (4.13)$$

$$\theta'' = \frac{Pr}{1 - \gamma Prf^2}(\gamma ff'\theta' - f\theta'), \quad (4.14)$$

$$\phi'' = -Scf\phi'. \quad (4.15)$$

subject to the boundary conditions

$$f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1, \quad \text{at} \quad \eta = 0, \tag{4.16}$$

$$f'(\eta) \to A, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \tag{4.17}$$

Let us denote

$$f = y_1, \quad f' = y_2, \quad f'' = y_3, \quad \theta = y_4, \quad \theta' = y_5, \quad \phi = y_6, \quad \phi' = y_7. \tag{4.18}$$

The nonlinear and coupled momentum, heat and concentration equations are transmuted into a system of seven first order instantaneous equations. The set of first Order ODEs along with the boundary conditions becomes

$$y_1' = y_2 \qquad y_1(0) = 0 \tag{4.19}$$

$$y_2' = y_3 \qquad y_2(0) = 1 \tag{4.20}$$

$$y_3' = \left(\frac{1}{1-\beta y_1^2}\right)\begin{bmatrix} -MA - A^2 + (M+K)y_2 + \\ {y_2}^2 - 2\beta y_1 y_2 y_3 - (1+\beta) y_1 y_3 \end{bmatrix} \qquad y_3(0) = s \tag{4.21}$$

$$y_4' = y_5 \qquad y_4(0) = 1 \tag{4.22}$$

$$y_5' = \frac{Pr}{1-\gamma Pr y_1^2}(\gamma y_1 y_2 y_5 - y_1 y_5) \qquad y_5(0) = t \tag{4.23}$$

$$y_6' = y_7 \qquad y_6(0) = 1 \tag{4.24}$$

$$y_7' = -Scy_1 y_7 \qquad y_7(0) = u. \tag{4.25}$$

For solving above system numerically, we replace the domain $[0, \infty)$ by the bounded domain $[0, \eta_\infty]$ where η_∞ is some suitable real number. In the above system of equations we have $y_3(\eta)$, $y_5(\eta)$ and $y_7(\eta)$ at $\eta = 0$ i.e., s, t and u are missing conditions and are to be chosen such that

$$y_2(\eta_\infty, s, t) \approx 0, \qquad y_4(\eta_\infty, s, t) \approx 0 \qquad \text{and} \qquad y_6(\eta_\infty, s, t) \approx 0.$$

4.4 Results and discussion

This section aims to examine the effect of different parameters M, K, A, β, Pr, γ and Sc (i.e., magnetic parameter, porosity parameter, stretching ratio parameter, Deborah number, prandtl number, non dimensional thermal relaxation time parameter and Schmidt number) on dimensionless velocity, temperature and concentration in the form of tables and graphs. Here, we include the conversation on numerical results obtained by shooting technique. Also the velocity, temperature and concentration profile is plotted in which the influence of different parameters is discussed.

To visualize the effect of different parameters numerical results are attained and are tabulated in Table 4.1 and 4.2. For the validity of present analysis, a comparison of present results with Mahapatra and Gupta [3] and Ishak et al. [4] is discussed in Tables 4.3 and 4.4. In Table 4.3 and 4.4 we have taken M, γ, K =0.

							$-f''(0)$		-$\theta'(0)$	
β	M	A	K	γ	Pr	Sc	bvp4c	Shooting	bvp4c	Shooting
0.3	0.1	0.1	0.1	0.5	1.0	2.0	1.1355041	1.1355031	0.6050838	0.6050837
0.5							1.1828328	1.1828336	0.5912941	0.5912943
0.7							1.229482	1.229481	0.577870	0.577870
	0.2						1.077498	1.077499	0.636992	0.636993
	0.5						0.798767	0.798768	0.745345	0.745348
	1.0						0.044135	0.044139	1.100055	1.100060
		0.2					1.077498	1.077499	0.636992	0.636993
		0.3					1.000720	1.000719	0.670242	0.670241
		0.4					0.906847	0.906848	0.729453	0.729455
			0.3				1.221272	1.221271	0.590773	0.590773
			0.5				1.299390	1.299391	0.600664	0.600664
			1.2				1.543654	1.543652	0.6758265	0.6758263
				0.2			1.135504	1.135503	0.580542	0.580542
				0.4			1.135504	1.135503	0.596509	0.596509
				0.8			1.133991	1.133992	0.634370	0.634373
					0.9		1.135504	1.135503	0.562295	0.562295
					0.72		1.135504	1.135503	0.481124	0.481124
					0.3		1.135504	1.135503	0.282923	0.282923
						1.5	1.135504	1.135503	0.605083	0.605083
						1.0	1.133991	1.133992	0.608772	0.608772
						0.5	1.133991	1.133992	0.608772	0.608772

TABLE 4.1: Numerical results of -$f''(0)$ and -$\theta'(0)$ for different values of β, M, A, K, γ, Pr and Sc.

							$-\phi'(0)$	
β	M	A	K	γ	Pr	Sc	bvp4c	Shooting
0.3	0.1	0.1	0.1	0.5	1.0	2.0	0.8849071	0.8849078
0.5							0.8717088	0.8717083
0.7							0.858567	0.858568
	0.2						0.909274	0.909273
	0.5						1.011166	1.011166
	1.0						1.336941	1.336940
		0.2					0.909274	0.909273
		0.3					0.937825	0.937826
		0.4					0.990562	0.990561
			0.3				0.868860	0.868861
			0.5				0.864571	0.864571
			1.2				0.891050	0.891051
				0.2			0.884907	0.884907
				0.4			0.884907	0.884907
				0.8			0.886967	0.886966
					0.9		0.884907	0.884907
					0.72		0.884907	0.884907
					0.3		0.884907	0.884907
						1.5	0.737047	0.737047
						1.0	0.582416	0.582415
						0.5	0.412136	0.412136

TABLE 4.2: Numerical results of $-\phi'(0)$ for different values of β, M, A, K, γ, Pr and Sc.

Pr	A	Mahapatra and Gupta [3]	present results	
			bvp4c	Shooting
1.0	0.1	0.603	0.603	0.603
	0.2	0.625	0.625	0.625
	0.5	0.692	0.692	0.692
1.5	0.1	0.777	0.777	0.777
	0.2	0.797	0.797	0.797
	0.5	0.863	0.864	0.864

TABLE 4.3: Comparison of $-\theta'(0)$ with Mahapatra and Gupta [3] for different values of Pr and A by taking $M = \gamma = K = 0$

A	Ishak et al. [4]	present results	
		bvp4c	Shooting
0.01	-0.9980	-0.9980	-0.9980
0.10	-0.9694	-0.9694	-0.9694
0.20	-0.9181	-0.9181	-0.9181
0.50	-0.6673	-0.66726	-0.66726
2.00	2.0175	2.0175	2.01749
3.00	4.7294	4.72928	4.72925

TABLE 4.4: Comparison of -$f''(0)$ with Ishak et al. [4] for different values of A by taking $M = \gamma = K = 0$

To observe the effects of different parameters on dimensionless velocity $f'(\eta)$, dimensionless temperature $\theta(\eta)$ and dimensionless concentration $\phi(\eta)$ graphs are plotted below. Figure 4.1 shows the impact of Deborah number β on the dimensionless velocity f'. This figure represents that velocity profile f' decreases for increasing values of Deboreh number β. The velocity profile $f'(\eta)$ for different values of porosity parameter K is plotted in Figure 4.2. It depicts that for increasing values of porosity parameter K velocity profile decreases. In general, rise in porosity parameter enlarge the porous layers of the flow which rise the velocity boundary layer thickness. Figure 4.3 shows the impact of Schmidt number Sc on concentration profile ϕ. The physical significance shows that increasing in Schmidt number Sc means decrease in molecular diffusion. The concentration is smaller for largest values of schmidt number Sc and is greater for smaller values of Sc. Thus, as the schmidt number Sc increases concentration ϕ decreases. This causes the concentration buoyancy effects to decrease, consequently there is a reduction in the fluid velocity. In Figure 4.4, the influence of thermal relaxation time parameter γ on temperature profile θ is represented. It is clear from fig that θ decreases when the thermal relaxation time parameter γ, the ratio of relaxation time and deformation time, increases. Figure 4.5 is the graphical representation which depicts the influence of Prandlet number Pr on dimensionless temperature. It is seen that temperature profile reduces for the rising estimations of Prandtl number. Greater the Prandltl number outcomes the lower thermal diffusivity. In this way increment in Prandtl number decreases conduction and henceforth the variation in thermal characteristics increases. Figure 4.6 is the graphical representation which shows the effect of magnetic parameter M on velocity profile f'. It depicts that rise in M , cause decrease in velocity profile, because increasing value of M has tendency to increase Lorentz force which yields more resistance to the transport phenomena. Figure 4.7 shows the influence of M on dimensionless temperature θ. It is clear from the fig that effect of magnetic parameter M on velocity profile and temperature profile is opposite. Figure 4.8 indicates how the Deborah number β affects the concentration profile ϕ. From this graph it is clear that for the larger values of Deborah number β concentration profile ϕ increases. As the elasticity parameter or Deborah number lessens the flow speed, it implies that less fluid is taken away at any given point bringing about the concentration profiles expanding. Figure 4.9 depicts the impact the Deborah number β on the temperature distribution θ. A diminishing in the stream-wise velocity component, can bring a decrease in the amount of heat transferred on the surface sheet. Correspondingly, a lessening in the transverse velocity component, suggests that the measure of new uid which is connected from the lower-temperature region outside the boundary layer and encouraged towards the sheet is reduced in this manner, diminishing the rate of warmth exchange. These two consequences on the velocity in a similar course fortify each other. In this manner, an epansion in the Deborah number expands the temperature distribution in the fluid as depicted in Figure 4.9. Figure 4.10 shows the impact of porosity parameter K on concentration profile ϕ. from this graph it is observed that for increasing values of porosity parameter K cause decrease in concentration. Figure 4.11 demonstrates the impact of porosity parameter K on temperature profile θ. An expansion in the porosity parameter is relied upon to diminish the quantity of heat transfer from the plate to the fluid, as recommended by fig. That is, an expansion in the porosity parameter expands temperature of the liquid at any given point over the sheet. In Figure 4.12, the effect of magnetic field parameter M on the concentration profile ϕ is displayed. From this graph it can be seen that for the bigger estimations of magnetic parameter M there is lessening in concentration ϕ. The reduction of flow velocity as the consequence of expanding the quality of the magnetic field makes the liquid fixation increment as less liquid is taken downstream at any given point.

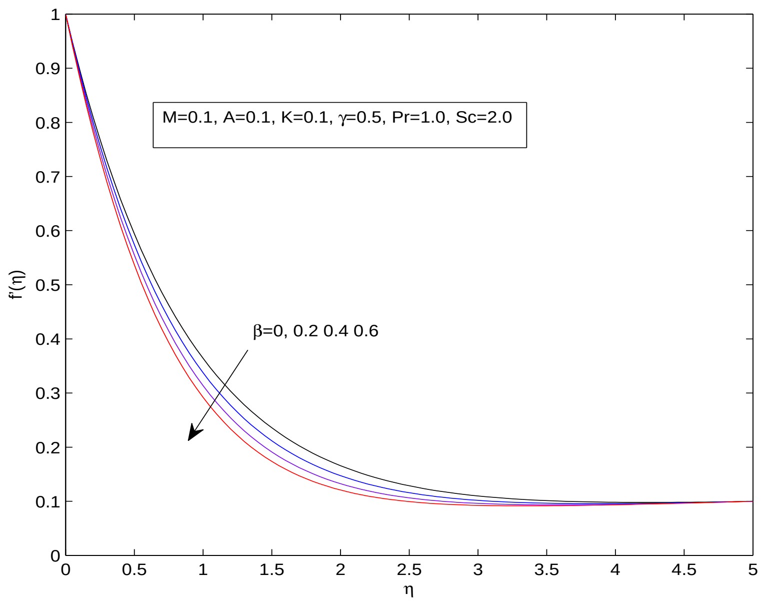

FIGURE 4.1: Impact of β on the dimensionless velocity f'

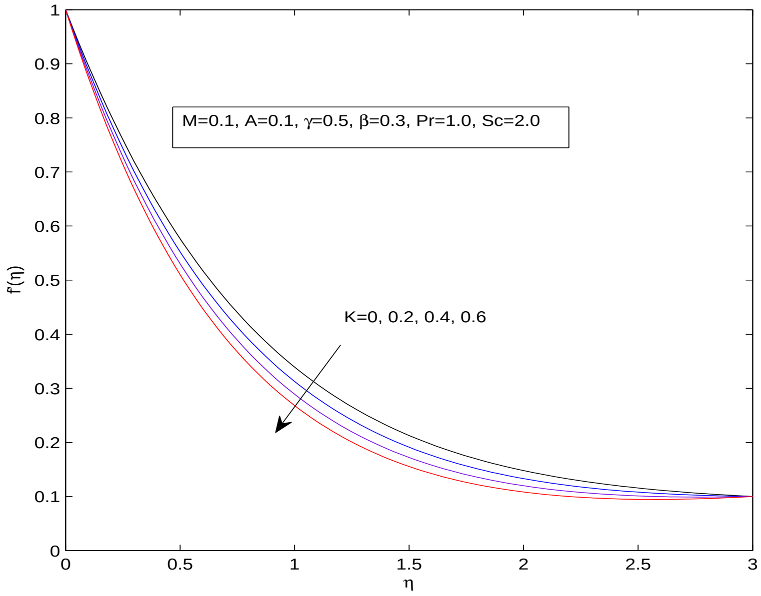

FIGURE 4.2: Influence of K on the dimensionless velocity f'

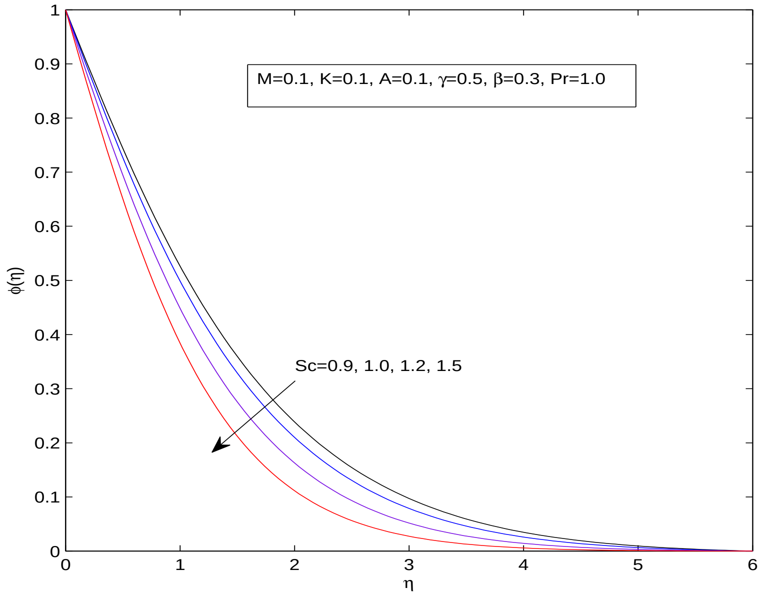

FIGURE 4.3: Effect of Sc on the dimensionless concentration ϕ

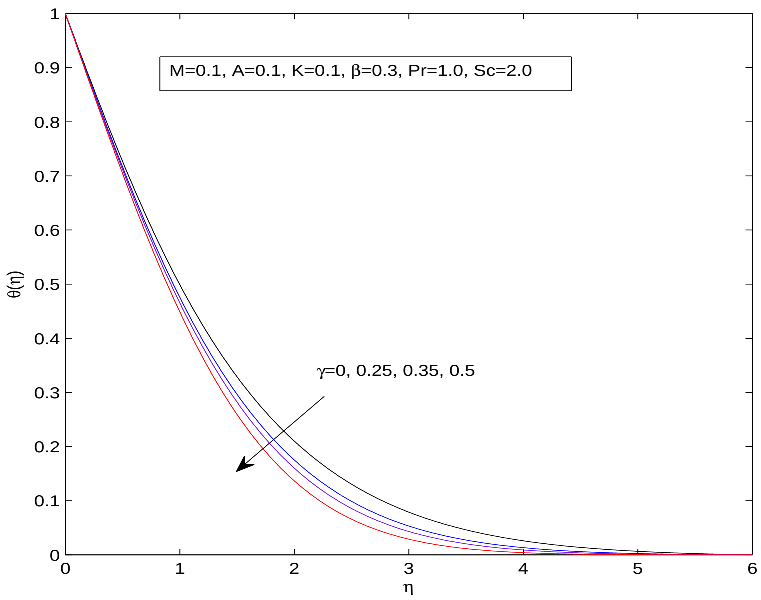

FIGURE 4.4: Influence of γ on the dimensionless temperature θ

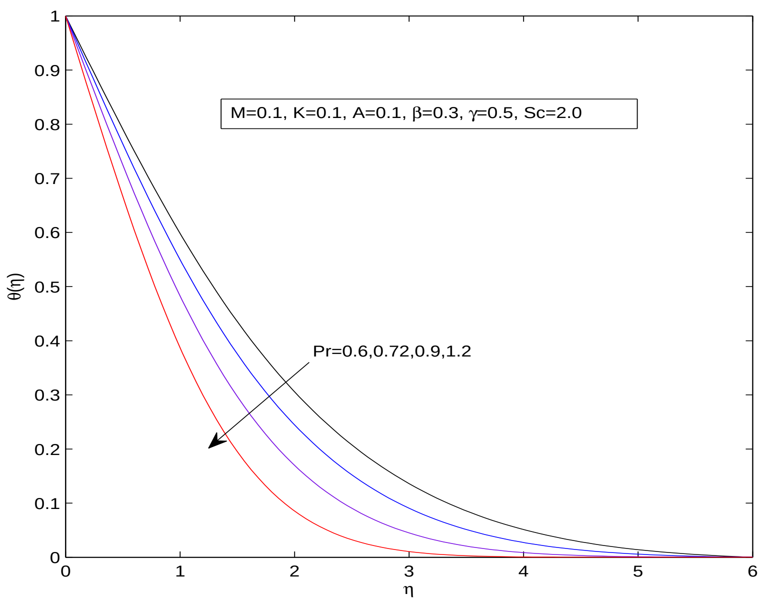

FIGURE 4.5: Influence of Pr on the dimensionless temperature θ

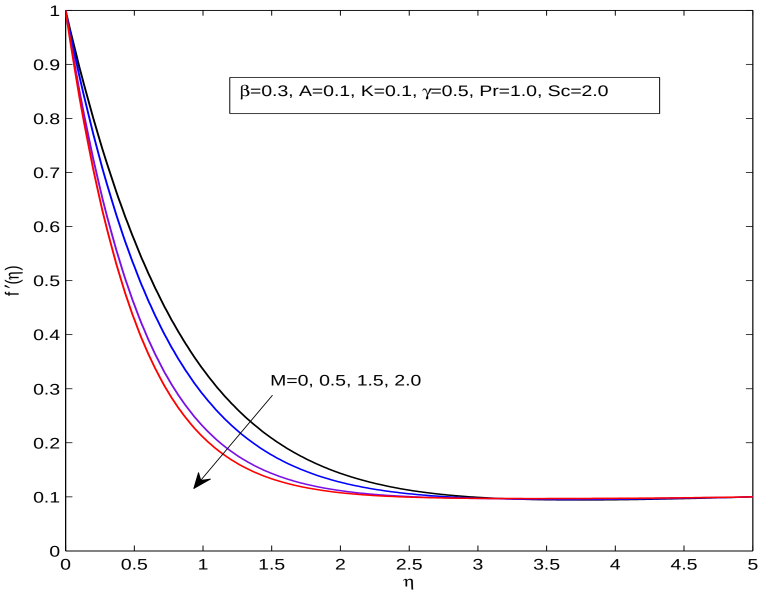

FIGURE 4.6: Influence of M on the dimensionless velocity f'

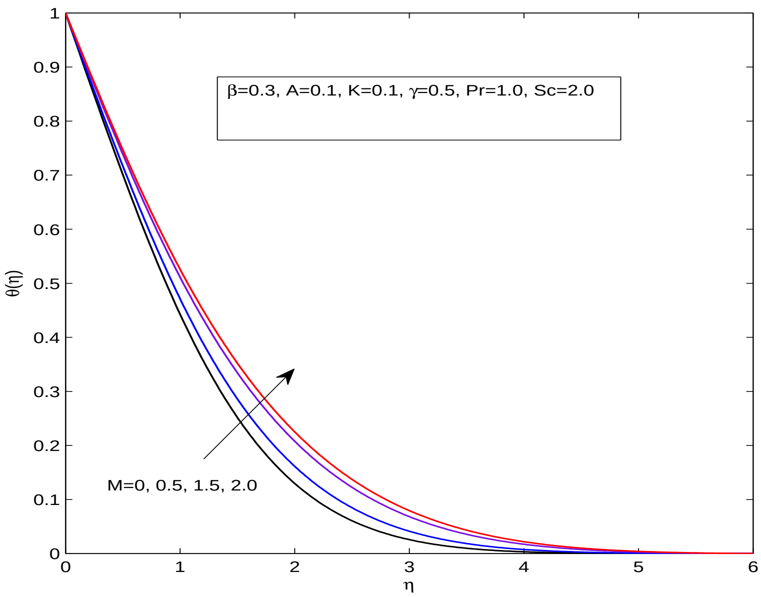

FIGURE 4.7: Influence of M on the dimensionless temperature θ

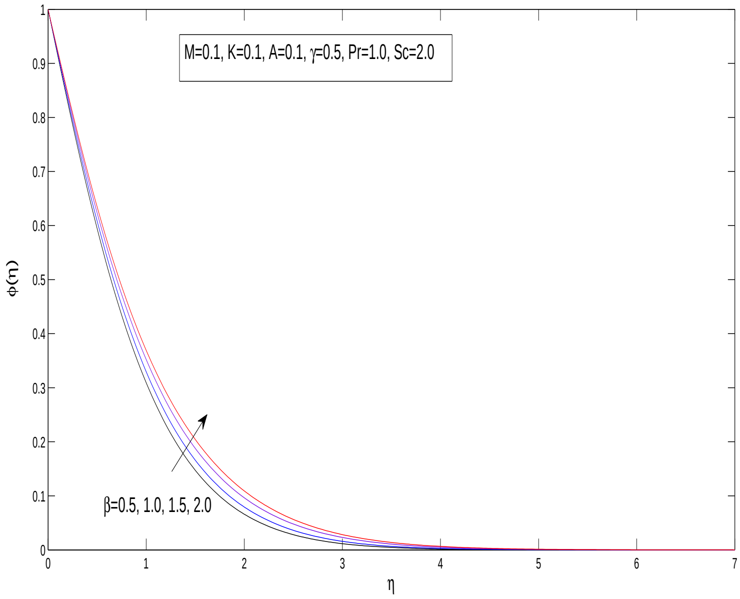

FIGURE 4.8: Effect of β on the dimensionless concentration ϕ

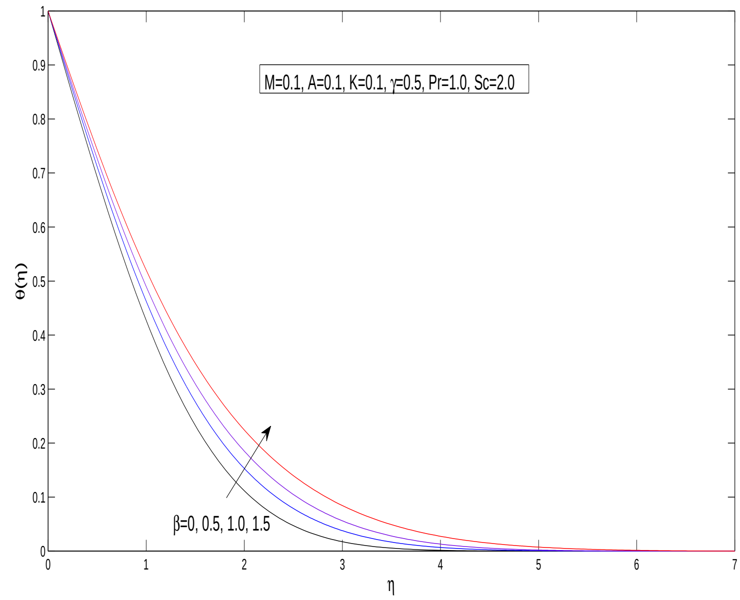

FIGURE 4.9: Effect of β on the dimensionless temperature θ

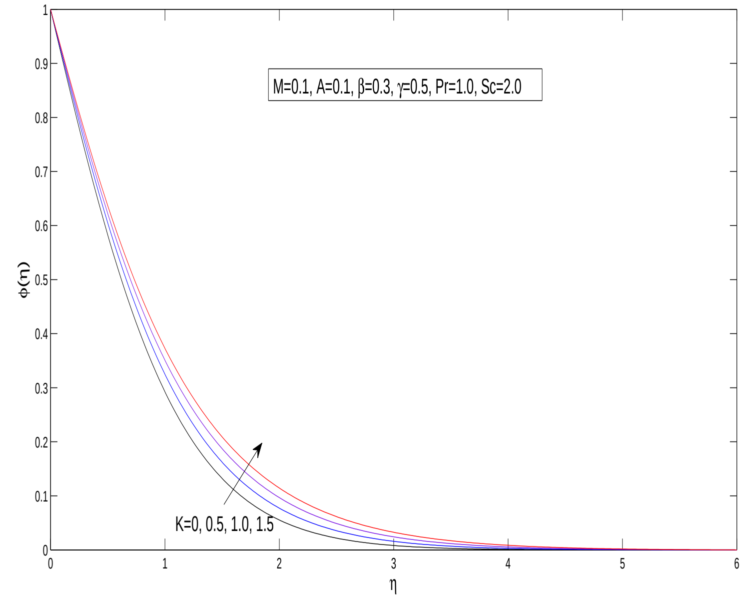

FIGURE 4.10: Influence of K on the dimensionless concentration ϕ

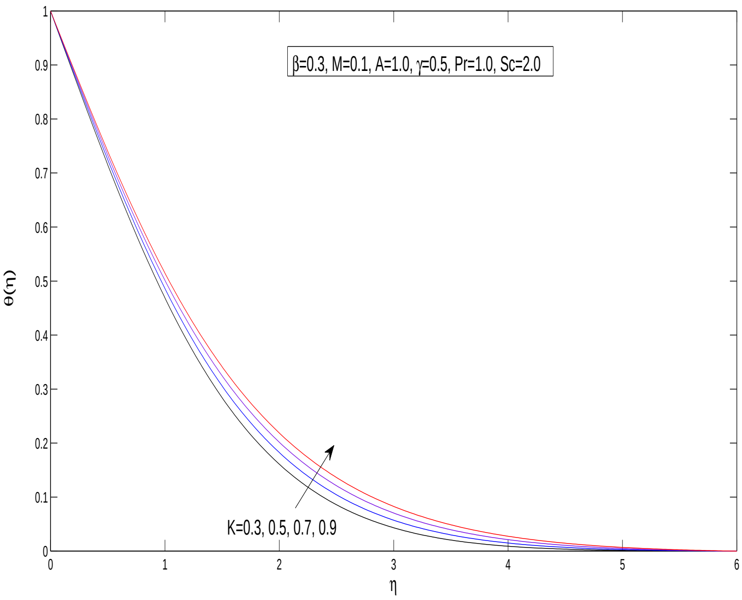

FIGURE 4.11: Influence of K on the dimensionless temperature θ

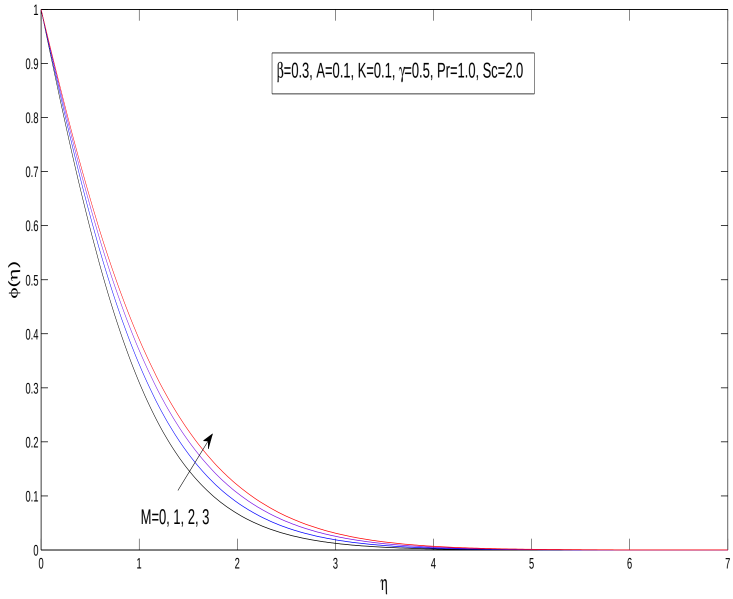

FIGURE 4.12: Influence of M on the dimensionless concentration ϕ

Chapter 5

Conclusion

In this research work, the study of incompressible, laminar, steady and two-dimensional magnetohydrodynamic flow of an "upper convected Maxwell fluid" past a semi-infinite porous plate due to thermal radiation and Joule heating by using "Cattaneo-Christov heat flux model" is discussed and is extended by considering stagnation point with concentration equation. The associated set of nonlinear partial differential equations (PDEs) of velocity, temperature and concentration are reduced by using a helpful similarity transformation. Numerical solution of these modeled ordinary differential equations (ODEs) is attained by using shooting technique. A numerical correlation has shown for different physical parameters influencing flow and heat transfer and found to be in excellent agreement with MATLAB built-in function bvp4c. Impact of different physical parameters such as magnetic parameter M, β the Deborah number, non dimensional thermal relaxation time parameter γ, Prandtl number Pr, radiational parameter is Rd, Eckert number Ec, porosity parameter K, schmidt number Sc on velocity, temperature and concentration profiles are discussed graphically and tabularly.

Conclusions which are obtained:

- Because of strong magnetic parameter M causes diminish in velocity and increament in temperature.
- With the increase in Deborah number β temperature increases, while the velocity decreases in the horizontal direction.
- Temperature profile rises while extending the radiation parameter and a same effect of Eckert number is seen on the temperature field.
- On temperature profile Prandtl number has decreasing effects.

- Velocity field f' decreases by enlarging M.
- Temperature field θ increases with an increase in M.
- Velocity filed f' decreases for increasing values of β.
- By increasing the values of Prandtl number results decrease in temperature θ.
- Increasing the values of schmidt number Sc causes decrease in concentration ϕ.

5.1 Future recommendations.

The present model has utilized many generalizations to emphasis on the principal effects of slip parameter, temperature dependent thermal conductivity and viscosity. A motivating area is to examine the impact of different nano particles, second order slip at the boundary, viscous dissipation and heat transfer of non-Newtonian fluids. Most likely there is a possibility for the trial chip away at such frameworks.

Bibliography

[1] K. Hiemenz. *Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszylinder.* PhD thesis, 1911.

[2] E. Eckert. *Die Berechnung des Wärmeübergangs in der laminaren Grenzschicht umströmter Körper.* VDI-Forschunhsheft, 1942.

[3] T. R. Mahapatra and A. S. Gupta. Heat transfer in stagnation-point flow towards a stretching sheet. *Heat and Mass transfer*, 38(6):517–521, 2002.

[4] A. Ishak, R. Nazar, and I. Pop. Mixed convection boundary layers in the stagnation-point flow towards a stretching vertical sheet. *Meccanica*, 41(5):509–518, 2006.

[5] T. Hayat, M. Mustafa, S. A. Shehzad, and S. Obaidat. Melting heat transfer in the stagnation-point flow of an upper-convected maxwell UCM fluid past a stretching sheet. *Int. J. Numerical Methods in Fluids*, 68(2):233–243, 2012.

[6] H. Alfvén. Existence of electromagnetic-hydrodynamic waves. *Nature*, 150(3805): 405–406, 1942.

[7] I. U. Mbeledogu and A. Ogulu. Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. *Int. J. Heat and Mass Transfer*, 50(9):1902–1908, 2007.

[8] D. C. Kesavaiah, P. V. Satyanarayana, and S. Venkataramana. Effects of the chemical reaction and radiation absorption on an unsteady mhd convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. *Int. J. Applied Mathematics and Mechanics*, 7(1):52–69, 2011.

[9] M. Modather and A. Chamka et al. An analytical study of MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium. *Turk. J. Engineering and Environmental Sciences*, 33(4):245–258, 2010.

[10] F. Mabood, W. A. Khan, and A. I. M. Ismail. MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration. *Chem. Eng. J*, 273:430–437, 2015.

[11] T. Hayat, S. Asad, M. Mustafa, and A. Alsaedi. MHD stagnation-point flow of Jeffrey fluid over a convectively heated stretching sheet. *Computers & Fluids*, 108: 179–185, 2015.

[12] M. Mustafa, J. A. Khan, T. Hayat, and A. Alsaedi. Sakiadis flow of Maxwell fluid considering magnetic field and convective boundary conditions. *AIP Advances*, 5 (2):027106, 2015.

[13] L. J. Crane. Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 21(4):645–647, 1970.

[14] T. C. Chiam. Hydromagnetic flow over a surface stretching with a power-law velocity. *Int. J. Engineering Science*, 33(3):429–435, 1995.

[15] S. J. Liao and I. Pop. Explicit analytic solution for similarity boundary layer equations. *Int. J. Heat and Mass Transfer*, 47(1):75–85, 2004.

[16] S. K. Khan and E. Sanjayanand. Viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet. *Int. J. Heat and Mass Transfer*, 48(8):1534–1542, 2005.

[17] T. Fang, F. L. Chia-fon, and J. Zhang. The boundary layers of an unsteady incompressible stagnation-point flow with mass transfer. *Int. J. Non-Linear Mechanics*, 46(7):942–948, 2011.

[18] J. Fourier. *Theorie analytique de la chaleur, par M. Fourier.* Chez Firmin Didot, père et fils, 1822.

[19] C. Cattaneo. Sulla conduzione del calore. In *Some Aspects of Diffusion Theory*, pages 485–485. Springer, 2011.

[20] C. I. Christov. On frame indifferent formulation of the Maxwell-Cattaneo model of finite-speed heat conduction. *Mechanics Research Communications*, 36(4):481–486, 2009.

[21] B. Straughan. Thermal convection with the Cattaneo-Christov model. *Int. J. Heat and Mass Transfer*, 53(1):95–98, 2010.

[22] V. Tibullo and V. Zampoli. A uniqueness result for the Cattaneo-Christov heat conduction model applied to incompressible fluids. *Mechanics Research Communications*, 38(1):77–79, 2011.

[23] J. A. Khan, M. Mustafa, T. Hayat, and A. Alsaedi. Numerical study of Cattaneo-Christov heat flux model for viscoelastic flow due to an exponentially stretching surface. *PloS one*, 10(9):e0137363, 2015.

[24] T. Hayat, M. Qasim, and S. Mesloub. Magnetohydrodynamic flow and heat transfer over permeable stretching sheet with slip conditions. *Int. J. Numerical Methods in Fluids*, 66(8):963–975, 2011.

[25] K. B. Pavlov. Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. *Magnitnaya Gidrodinamika*, 4(1):146–147, 1974.

[26] W. A. Khan and A. Aziz. Natural convection flow of a nanofluid over a vertical plate with uniform surface heat flux. *Int. J. Thermal Sciences*, 50(7):1207–1214, 2011.

[27] W. A. Khan and I. Pop. Boundary-layer flow of a nanofluid past a stretching sheet. *Int. J. Heat and Mass Transfer*, 53(11):2477–2483, 2010.

[28] O. D. Makinde and A. Aziz. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *Int. J. Thermal Sciences*, 50(7):1326–1332, 2011.

[29] A. V. Kuznetsov and D. A. Nield. Natural convective boundary-layer flow of a nanofluid past a vertical plate. *Int. J. Thermal Sciences*, 49(2):243–247, 2010.

[30] N. A. Yacob, A. Ishak, I. Pop, and K. Vajravelu. Boundary layer flow past a stretching/shrinking surface beneath an external uniform shear flow with a convective surface boundary condition in a nanofluid. *Nanoscale research letters*, 6(1):314, 2011.

[31] S. Choi. Enhancing thermal conductivity of fluids with nanoparticles in developments and applications of non-Newtonian flows (ed. da. siginer et hp. wang) pp. 99-105, New York: Am. Soc. *Mech. Eng*, 1995.

[32] B. V. Derjaguin and Y. Yalamov. Theory of thermophoresis of large aerosol particles. *J. Colloid science*, 20(6):555–570, 1965.

[33] A. J. Chamkha and C. Issac. Effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface. *Int. J. Numerical Methods for Heat & Fluid Flow*, 10(4):432–449, 2000.

[34] R. Tsai. A simple approach for evaluating the effect of wall suction and thermophoresis on aerosol particle deposition from a laminar flow over a flat plate. *Int. C. Heat and Mass Transfer*, 26(2):249–257, 1999.

[35] K. Vajravelu. Viscous flow over a nonlinearly stretching sheet. *Applied mathematics and computation*, 124(3):281–288, 2001.

[36] B. Gebhart and J. Mollendorf. Viscous dissipation in external natural convection flows. *J. Fluid mechanics*, 38(01):97–107, 1969.

[37] H. M. Duwairi. Viscous and Joule heating effects on forced convection flow from radiate isothermal porous surfaces. *Int. J. Numerical Methods for Heat & Fluid Flow*, 15(5):429–440, 2005.

[38] S. Shah, S. Hussain, and M. Sagheer. MHD effects and heat transfer for the UCM fluid along with Joule heating and thermal radiation using Cattaneo-Christov heat flux model. *AIP Advances*, 6(8):085103, 2016.

[39] S. S. Elnashaie and F. Uhlig. *Numerical techniques for chemical and biological engineers using MATLAB®: A simple bifurcation approach.* Springer Science & Business Media, 2007.

[40] G. Bar-Meir. Basics of fluid mechanics. *Chicago*, 133323:1700, 2013.

[41] C. I. Christov. On frame indifferent formulation of the maxwell–cattaneo model of finite-speed heat conduction. *Mechanics Research Communications*, 36(4):481–486, 2009.

[42] M. Renardy. High Weissenberg number boundary layers for the upper convected Maxwell fluid. *J. Non-Newtonian Fluid Mechanics*, 68(1):125–132, 1997.

[43] N. Fidya and M. Noor. Analysis for MHD flow of a maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction. *World Acad. Sci., Eng. Technol*, 64:1019–1023, 2012.